**INSTRUCTION:**
Answer **ONE (1)** question from each section (A, B and C) and answer **ONE (1)** question from any section that has not been answered.

***ARAHAN :***
*Jawab **SATU (1)** soalan daripada setiap bahagian (A, B dan C) dan Jawab **SATU(1)** lagi soalan yang belum dijawab dari mana-mana bahagian.*

**SECTION A**
***BAHAGIAN A***

**QUESTION 1**
***SOALAN 1***

CLO1
C1

(a) By using definition of Hyperbolic Functions, find the value of

*Dengan menggunakan definisi Fungsi Hiperbola, cari nilai bagi*

i. $\sinh\left(-\sqrt{3}\right)$ [2 marks] [*2 markah*]

ii. $\tanh 3.42$ [2 marks] [*2 markah*]

iii. $\operatorname{cosech}\frac{3}{4}$ [2 marks] [*2 markah*]

CLO1
C1

(b) If $y^3 = 0.69x\cosh\frac{0.9z}{x}$, determine the value of $y$ when $x = 40$ and $z = 138$.

*Jika* $y^3 = 0.69x\,kosh\frac{0.9z}{x}$, *tentukan nilai bagi* $y$ *apabila* $x = 40$ *dan* $z = 138$

[5 marks]
[*5 markah*]

CLO1
C2

(c) Prove that $\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y$.

*Buktikan bahawa* $kosh(x+y) = kosh\,x\ kosh\,y + \sinh x\sinh y$.

[7 marks]
[*7 markah*]



SULIT

**POLITEKNIK**
**Jabatan Pengajian Politeknik**

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENGAJIAN POLITEKNIK
KEMENTERIAN PENDIDIKAN MALAYSIA

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

**SESI JUN 2013**

**BA601: ENGINEERING MATHEMATICS 5**

**TARIKH : 22 OKTOBER 2013**
**TEMPOH : 2 JAM (2.30 PM - 4.30 PM)**

Kertas ini mengandungi **LAPAN (8)** halaman bercetak.
Bahagian A: Struktur (2 soalan) – Jawab **SATU (1)** Soalan sahaja
Bahagian B: Struktur (2 soalan) – Jawab **SATU (1)** Soalan sahaja
Bahagian C: Struktur (2 soalan) – Jawab **SATU (1)** Soalan sahaja
DAN Jawab **SATU (1)** Soalan Dari Mana-mana Bahagian A/ B/ C yang belum dijawab.
Dokumen sokongan yang disertakan : Formula

**JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN**

(CLO yang tertera hanya sebagai rujukan)

CLO1 C2

(d) Sketch a quadrant graph and find the principal value for the following functions.

*Lakarkan graf sukuan dan dapatkan nilai utama bagi fungsi-fungsi berikut:*

i. $\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

[4 marks]
[4 *markah*]

ii. $\sin^{-1}(-0.9659)$

[4 marks]
[4 *markah*]





CLO1 C2

(d) Complete the table below for equation $y = \cosh(x) + 2$. Then, sketch the graph in the range given, as $-2 \le x \le 2$.

*Lengkapkan jadual di bawah bagi persamaan* $y = kosh(x) + 2$. *Seterusnya lakarkan graf yang terhasil, bagi julat* $-2 \le x \le 2$.

| $x$ | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | | | | | |

[7 marks]
[7 *markah*]

## QUESTION 2
## *SOALAN 2*

CLO1 C1

(a) Calculate the value for each of the following functions.
*Kirakan nilai bagi setiap fungsi yang berikut.*

i. $\cosh\left(\dfrac{2}{3}\right)$

[2 marks]
[2 *markah*]

ii. $\tanh^{-1}(0.623)$

[2 marks]
[2 *markah*]

iii. $\operatorname{cosech}^{-1}(3)$

[3 marks]
[3 *markah*]

CLO1 C1

(b) If $y^2 = 0.5x\tanh 0.04x$, find the value of $y$ if $x = 10$

*Jika* $y^2 = 0.5x\tanh 0.04x$, *dapatkan nilai y jika* $x = 10$

[4 marks]
[4 *markah*]

CLO1 C3

(c) Prove that $\cosh^2 x - \sinh^2 x = 1$.

*Buktikan bahawa* $\text{kosh}^2 x - \sinh^2 x = 1$.

[6 marks]
[6 *markah*]

QUESTION 4
*SOALAN 4*

CLO2
C2

(a) Determine.

*Tentukan.*

i. $\int x \sinh x^2 dx$

[4 marks]
[4 *markah*]

ii. $\int \frac{dx}{4+25x^2}$

[5 marks]
[5 *markah*]

iii. $\int \frac{dx}{\sqrt{36+4x^2}}$

[4 marks]
[4 *markah*]

CLO2
C3

(b) Solve.

*Selesaikan.*

i. $\int_1^2 (\cosh 2x) dx$

[6 marks]
[6 *markah*]

ii. $\int_0^2 \frac{dx}{2+9x^2}$

[6 marks]
[6 *markah*]





SECTION B
*BAHAGIAN B*

QUESTION 3
*SOALAN 3*

CLO2
C2

(a) Differentiate the following functions with respect to $x$.

*Bezakan fungsi-fungsi yang berikut terhadap* $x$.

i. $y = \sin^{-1}(2x)$

[4 marks]
[4 *markah*]

ii. $y = \tanh(\ln 2x)$

[4 marks]
[4 *markah*]

iii. $y = 2x \cosh^{-1}(x)$

[4 marks]
[4 *markah*]

CLO2
C2

(b) Determine $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x^2}$ for $z = 6xy + \cos xy$.

*Tentukan* $\frac{\partial z}{\partial x}$ *dan* $\frac{\partial^2 z}{\partial x^2}$ *untuk* $z = 6xy + \text{kos}xy$.

[8 marks]
[8 *markah*]

CLO2
C2

(c) Use **implicit differentiation method** to determine the derivative for the following functions.

*Gunakan* ***kaedah pembezaan tersirat****, tentukan pembezaan bagi fungsi yang berikut.*

$x^2 + y^2 - 4x - 8y + 12 = 0$

[5 marks]
[5 *markah*]

**QUESTION 6**
*SOALAN 6*

CLO3 C2 (a) Find the general solution of second order differential equation below.

*Cari penyelesaian am bagi persamaan pembezaan peringkat kedua di bawah.*

i. $\dfrac{d^2y}{dx^2}+5\dfrac{dy}{dx}+6y$ [4 marks] [*4 markah*]

ii. $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+4y$ [5 marks] [*5 markah*]

iii. $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+7y$ [7 marks] [*7 markah*]

CLO3 C4 (b) Solve the following differential equation.

*Selesaikan persamaan pembezaan berikut.*

$3x\dfrac{dy}{dx}=x+4y$ [9 marks] [*9 markah*]

**SOALAN TAMAT**





## SECTION C
## *BAHAGIAN C*

**QUESTION 5**
*SOALAN 5*

CLO3 C2 (a) Form a differential equation for the function below.

*Bentukkan persamaan pembezaan bagi fungsi di bawah.*

$$y=Ax^2+4Bx$$

[8 marks] [*8 markah*]

CLO3 C3 (b) Determine the general solution of the following differential equations.

*Tentukan penyelesaian am bagi persamaan pembezaan yang berikut.*

i. $\dfrac{dy}{dx}=2x+xy$ [5 marks] [*5 markah*]

ii. $x\dfrac{dy}{dx}=x^2-3x$ [5 marks] [*5 markah*]

iii. $\left(\dfrac{2y}{x^2+1}\right)\dfrac{dy}{dx}=y^3$ [7 marks] [*7 markah*]

| DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS (where $u$ is a function of $x$) | INTEGRALS INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS (where $u$ is a function of $x$) |
|---|---|
| $\frac{d}{dx}(\sin^{-1}u) = \frac{1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$ ; $\lvert u\rvert < 1$ | $\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\left(\frac{u}{a}\right)+C$ ; $\lvert u\rvert < a$ |
| $\frac{d}{dx}(\cos^{-1}u) = \frac{-1}{\sqrt{1-u^2}}\cdot\frac{du}{dx}$ ; $\lvert u\rvert < 1$ | $\int \frac{-1}{\sqrt{a^2-u^2}}du = \cos^{-1}\left(\frac{u}{a}\right)+C$ ; $\lvert u\rvert < a$ |
| $\frac{d}{dx}(\tan^{-1}u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$ | $\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$ |
| $\frac{d}{dx}(\csc^{-1}u) = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}}\cdot\frac{du}{dx}$ ; $\lvert u\rvert > 1$ | $\int \frac{-1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\csc^{-1}\left(\frac{u}{a}\right)+C$ ; $\lvert u\rvert > a$ |
| $\frac{d}{dx}(\sec^{-1}u) = \frac{1}{\lvert u\rvert\sqrt{u^2-1}}\cdot\frac{du}{dx}$ ; $\lvert u\rvert > 1$ | $\int \frac{1}{\lvert u\rvert\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right)+C$ ; $\lvert u\rvert > a$ |
| $\frac{d}{dx}(\cot^{-1}u) = \frac{-1}{1+u^2}\cdot\frac{du}{dx}$ | $\int \frac{-1}{a^2+u^2}du = \frac{1}{a}\cot^{-1}\left(\frac{u}{a}\right)+C$ |

| DERIVATIVES OF INVERSE HYPERBOLIC FUNCTIONS (where $u$ is a function of $x$) | INTEGRALS INVOLVING INVERSE HYPERBOLIC FUNCTIONS (where $u$ is a function of $x$) |
|---|---|
| $\frac{d}{dx}(\sinh^{-1}u) = \frac{1}{\sqrt{u^2+1}}\cdot\frac{du}{dx}$ | $\int \frac{1}{\sqrt{a^2+u^2}}du = \sinh^{-1}\left(\frac{u}{a}\right)+C$ ; $a > 0$ |
| $\frac{d}{dx}(\cosh^{-1}u) = \frac{1}{\sqrt{u^2-1}}\cdot\frac{du}{dx}$ ; $\lvert u\rvert > 1$ | $\int \frac{1}{\sqrt{u^2-a^2}}du = \cosh^{-1}\left(\frac{u}{a}\right)+C$ ; $u > a > 0$ |
| $\frac{d}{dx}(\tanh^{-1}u) = \frac{1}{1-u^2}\cdot\frac{du}{dx}$ ; $\lvert u\rvert < 1$ | $\int \frac{1}{a^2-u^2}du = \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right)+C$ ; if $\lvert u\rvert < a^2$ |
| $\frac{d}{dx}(\operatorname{csch}^{-1}u) = \frac{-1}{\lvert u\rvert\sqrt{1+u^2}}\cdot\frac{du}{dx}$ ; $u \neq 0$ | $\int \frac{1}{u\sqrt{u^2+a^2}}du = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{u}{a}\right)+C$ |
| $\frac{d}{dx}(\operatorname{sech}^{-1}u) = \frac{-1}{\lvert u\rvert\sqrt{1-u^2}}\cdot\frac{du}{dx}$ ; $0 < u < 1$ | $\int \frac{1}{u\sqrt{a^2-u^2}}du = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right)+C$ ; $0 < u < a$ |
| $\frac{d}{dx}(\coth^{-1}u) = \frac{1}{1-u^2}\cdot\frac{du}{dx}$ ; $\lvert u\rvert > 1$ | $\int \frac{1}{u^2-a^2}du = \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right)+C$ ; if $u^2 > a^2$ |





| TRIGONOMETRIC IDENTITIES | HYPERBOLIC IDENTITIES |
|---|---|
| $\cos^2 x + \sin^2 x = 1$ | $\cosh^2 x - \sinh^2 x = 1$ |
| $1 + \tan^2 x = \sec^2 x$ | $1 - \tanh^2 x = \operatorname{sech}^2 x$ |
| $\cot^2 x + 1 = \csc^2 x$ | $\coth^2 x - 1 = \operatorname{csch}^2 x$ |
| $\sin 2x = 2\sin x\cos x$ | $\sinh 2x = 2\sinh x\cosh x$ |
| $\cos 2x = \cos^2 x - \sin^2 x$ | $\cosh 2x = \cosh^2 x + \sinh^2 x$ |
| $\cos 2x = 1 - 2\sin^2 x$ | $\cosh 2x = 1 + 2\sinh^2 x$ |
| $\cos 2x = 2\cos^2 x - 1$ | $\cosh 2x = 2\cosh^2 x - 1$ |
| $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ | $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$ |
| $\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$ | $\sinh(x \pm y) = \sinh x\cosh y \pm \cosh x\sinh y$ |
| $\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$ | $\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$ |
| $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$ | $\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$ |

| RECIPROCAL TRIGONOMETRIC IDENTITIES | RECIPROCAL HYPERBOLIC IDENTITIES |
|---|---|
| $\csc x = \frac{1}{\sin x}$ | $\operatorname{csch} x = \frac{1}{\sinh x}$ |
| $\sec x = \frac{1}{\cos x}$ | $\operatorname{sech} x = \frac{1}{\cosh x}$ |
| $\cot x = \frac{1}{\tan x}$ | $\coth x = \frac{1}{\tanh x}$ |

| HYPERBOLIC FUNCTIONS | INVERSE HYPERBOLIC FUNCTIONS |
|---|---|
| $\sinh x = \frac{e^x - e^{-x}}{2}$ | $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$ ; $-\infty < x < \infty$ |
| $\cosh x = \frac{e^x + e^{-x}}{2}$ | $\cosh^{-1} x = \pm\ln\left(x + \sqrt{x^2-1}\right)$ ; $x \geq 1$ |
| $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ | $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ ; $\lvert x\rvert < 1$ |
| $\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$ ; $x \neq 0$ | $\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{\lvert x\rvert}\right)$ ; $x \neq 0$ |
| $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$ | $\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$ ; $0 < x \leq 1$ |
| $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ ; $x \neq 0$ | $\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$ ; $\lvert x\rvert > 1$ |

## SOLUTION FOR FIRST ORDER DIFFERENTIAL EQUATION

1. Direct Integration:

$$\frac{dy}{dx} = f(x)$$

2. Separating The Variables:

$$\frac{dy}{dx} = -\frac{f(x)}{f(y)}$$

3. Substitution $y = vx$ (Homogenous Equations)

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

4. Integrating Factors (Linear Equations)

$$y.IF = \int Q.IF\,dx$$

Where $IF = e^{\int P dx}$

### LOGARITHMIC

$a = e^{\ln a}$

$a^x = e^{x \ln a}$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

## GENERAL SOLUTION FOR SECOND ORDER DIFFERENTIAL EQUATION

Equation of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0.$

1. Real and different roots

$$y = Ae^{m_1x} + Be^{m_2x}$$

2. Real and equal roots

$$y = e^{m_1x}(A + Bx)$$

3. Complex roots

$$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

Equation of the form $\frac{d^2y}{dx^2} \pm n^2y = 0.$

1. $\frac{d^2y}{dx^2} + n^2y = 0.$

$$y = A\cos nx + B\sin nx$$

2. $\frac{d^2y}{dx^2} - n^2y = 0.$

$$y = A\cosh nx + B\sinh nx$$





| BASIC DERIVATIVES (where $u$ is a function of $x$) | BASIC INTEGRALS (where $u$ is a function of $x$) |
|---|---|
| $\frac{d}{dx}(k) = 0$ ; $k$ = constant | $\int (k)\,du = ku + C$ ; $k$ = constant |
| $\frac{d}{dx}(u^n) = nu^{n-1}$ | $\int (u^n)\,du = \frac{u^{n+1}}{n+1} + C$ ; $n \neq -1$ |
| $\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$ | $\int (e^u)\,du = \frac{e^u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\ln\lvert u\rvert) = \frac{1}{u}\cdot\frac{du}{dx}$ | $\int \left(\frac{1}{u}\right) du = \frac{\ln u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$ | $\int \sin u\,du = -\frac{\cos u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\cos u) = -\sin u \cdot \frac{du}{dx}$ | $\int \cos u\,du = \frac{\sin u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$ | $\int \sec^2 u\,du = \frac{\tan u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\csc u) = -\csc u \cot u \cdot \frac{du}{dx}$ | $\int \csc u \cot u\,du = -\frac{\csc u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\sec u) = \sec u \tan u \cdot \frac{du}{dx}$ | $\int \sec u \tan u\,du = \frac{\sec u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\cot u) = -\csc^2 u \cdot \frac{du}{dx}$ | $\int \csc^2 u\,du = -\frac{\cot u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\sinh u) = \cosh u \cdot \frac{du}{dx}$ | $\int \sinh u\,du = \frac{\cosh u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\cosh u) = \sinh u \cdot \frac{du}{dx}$ | $\int \cosh u\,du = \frac{\sinh u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\tanh u) = \text{sech}^2 u \cdot \frac{du}{dx}$ | $\int \text{sech}^2 u\,du = \frac{\tanh u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\text{csch}\, u) = -\text{csch}\, u \coth u \cdot \frac{du}{dx}$ | $\int \text{csch}\, u \coth u\,du = -\frac{\text{csch}\, u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\text{sech}\, u) = -\text{sech}\, u \tanh u \cdot \frac{du}{dx}$ | $\int \text{sech}\, u \tanh u\,du = -\frac{\text{sech}\, u}{\frac{du}{dx}} + C$ |
| $\frac{d}{dx}(\coth u) = -\text{csch}^2 u \cdot \frac{du}{dx}$ | $\int \text{csch}^2 u\,du = -\frac{\coth u}{\frac{du}{dx}} + C$ |